Sādhanā Vol. 27, Part 5, October 2002, pp. 493–506. © Printed in India

Reflection of plane micropolar viscoelastic waves at a loosely bonded solid–solid interface

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MS received 25 April 2000; revised 15 January 2002

Abstract. A solution of the field equations governing small motions of a micropolar viscoelastic solid half-space is employed to study the reflection and transmission of plane waves at a loosely bonded interface between two dissimilar micropolar viscoelastic solid half-spaces. The amplitude ratios for various reflected and refracted waves are computed for a particular model for different values of bonding parameter. The variations of these amplitude ratios with the angle of incidence are shown graphically. Effects of bonding parameter and viscosity on the amplitude ratios are shown.

Keywords. Micropolar viscoelastic solid; micropolar viscoelastic waves; amplitude ratios; bonding parameter.

1. Introduction

The theory of micropolar elasticity introduced and developed by Eringen (1968) has aroused much interest in recent years due to its possible utility in investigating deformation properties of solids, for which the purely elastic theory is inadequate. This theory is believed to be particularly useful for investigating materials consisting of bar-like molecules which exhibit microrotational effects, and which can also support body and surface couples. Examples of such materials are wood, fibres and corpuscles. Furthermore, the micropolar elastic model is more realistic than the purely elastic theory for studying geophysical problems and also nondestructive testing of solids. Various problems of waves and vibrations of micropolar elastic solids are discussed by several researchers. Notable among them are Smith (1967), Parfitt & Eringen (1969), Ariman (1972), Tomar & Gogna (1992, 1995), Tomar & Kumar (1995) etc.

The linear theory of micropolar viscoelasticity was developed by Eringen (1967). McCarthy & Eringen (1969) discussed the propagation conditions and growth equations, that govern the propagation of waves in micropolar viscoelasticity. They also studied the couplings between the discontinuities in the macroscopic and microscopic fields. Recently, Singh (2000) studied

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a problem on reflection and transmission of plane harmonic waves at an interface between liquid and micropolar viscoelastic solid with stretch.

In the problems of reflection and refraction of seismic waves at the interface between two elastic half-spaces, it is usually assumed that the half-spaces are in welded contact. However, the presence of liquid in the porous skeleton may weaken the welded contact at the interface. Hence it is reasonable to assume that a very thin layer of viscous liquid may be present at the interface and cause the two media to be loosely bonded. Murty (1976) discussed the reflection, refraction and attenuation of elastic waves at a loosely bonded interface between two elastic soild half-spaces by assuming that the interface behaves like a dislocation which preserves the continuity of traction allowing a finite amount of slip and derived the cases of welded contact and ideally smooth interfaces as particular cases. Kumar & Singh (1997) discussed a problem on reflection and transmission of elastic waves at the loosely bonded interface between an elastic solid half-space and a micropolar elastic solid half-space.

A more realistic model of the earth's crust is considered to study the reflection and transmission of plane elastic waves at a loosely bonded interface between two dissimilar micropolar viscoelastic solid half-spaces. This problem is of geophysical interest, particularly in investigations concerned with earthquakes and other phenomenon in seismology. The results of some earlier works (Murty 1976; Kumar & Singh 1997) follow as particular cases of the more general results presented in this paper.

2. Field equations and their solutions

Following Eringen (1967), the constitutive equations and field equations of a micropolar viscoelastic solid in the absence of body forces and body couples, can be written as

$$t_{kl} = \lambda u_{r,r}\delta_{kl} + \mu(u_{k,l} + u_{l,k}) + \kappa(u_{l,k} - \varepsilon_{klr}\phi_r), \tag{1}$$

$$m_{kl} = \alpha\phi_{r,r}\delta_{kl} + \beta\phi_{k,l} + \gamma\phi_{l,k}, \tag{2}$$

and

$$(c_1^2 + c_3^2)\nabla(\nabla\cdot\boldsymbol{u}) - (c_2^2 + c_3^2)\nabla\times(\nabla\times\boldsymbol{u}) + c_3^2\nabla\times\boldsymbol{\phi} = \ddot{\boldsymbol{u}}, \tag{3}$$

$$(c_4^2 + c_5^2)\nabla(\nabla\cdot\boldsymbol{\phi}) - c_4^2\nabla\times(\nabla\times\boldsymbol{\phi}) + \omega_o^2\nabla\times\boldsymbol{u} - 2\omega_o^2\boldsymbol{\phi} = \ddot{\boldsymbol{\phi}}, \tag{4}$$

where

$$\begin{aligned}
&c_1^2 = (\lambda + 2\mu)/\rho, && c_2^2 = \mu/\rho, && c_3^2 = \kappa/\rho,\\
&c_4^2 = \gamma/\rho j, && c_5^2 = (\alpha + \beta)/\rho j, && \omega_o^2 = c_3^2/j = \kappa/\rho j,\\
&\lambda = \lambda^* + \lambda_v^*(\partial/\partial t), && \mu = \mu^* + \mu_v^*(\partial/\partial t), && \kappa = \kappa^* + \kappa_v^*(\partial/\partial t),\\
&\alpha = \alpha^* + \alpha_v^*(\partial/\partial t), && \beta = \beta^* + \beta_v^*(\partial/\partial t), && \gamma = \gamma^* + \gamma_v^*(\partial/\partial t),\\
&\nabla = \hat{i}(\partial/\partial x) + \hat{k}(\partial/\partial z), && &&
\end{aligned} \tag{5}$$

$\lambda^*, \mu^*, \kappa^*, \alpha^*, \beta^*, \gamma^*, \lambda_v^*, \mu_v^*, \kappa_v^*, \alpha_v^*, \beta_v^*$ and γ_v^* are material constants, ρ is the density and j the rotational inertia. $\boldsymbol{u}$ and $\boldsymbol{\phi}$ are displacement and microrotation vectors respectively. Superposed dots on the right hand side of (3) and (4) denote the second partial derivative with respect to time.

Taking $\boldsymbol{u} = (u_1, 0, u_3)$ and $\boldsymbol{\phi} = (0, \boldsymbol{\phi}_2, 0)$ and introducing potentials $\phi(x, z, t)$ and $\psi(x, z, t)$ which are related to displacement components as

$$u_1 = (\partial\phi/\partial x) + (\partial\psi/\partial z), \quad u_3 = (\partial\phi/\partial z) - (\partial\psi/\partial x), \tag{6}$$

Substituting the displacement components given by (6) in the (3) and (4), we obtain

$$\left(\nabla^2 - \frac{1}{(c_1^2+c_3^2)}\frac{\partial^2}{\partial t^2}\right)\phi = 0, \tag{7}$$

$$\left(\nabla^2 - \frac{1}{(c_2^2+c_3^2)}\frac{\partial^2}{\partial t^2}\right)\psi - p\phi_2 = 0, \tag{8}$$

$$\left(\nabla^2 - 2q - \frac{1}{c_4^2}\frac{\partial^2}{\partial t^2}\right)\phi_2 + q\nabla^2\psi = 0, \tag{9}$$

where

$$p = \mu/(\mu+\kappa), q = \kappa/\gamma. \tag{10}$$

We assume time variation as

$$\begin{aligned} \phi(x,z,t) &= \overline{\phi}(x,z)\exp(i\omega t), \\ \psi(x,z,t) &= \overline{\psi}(x,z)\exp(i\omega t), \\ \phi_2(x,z,t) &= \overline{\phi}_2(x,z)\exp(i\omega t). \end{aligned} \tag{11}$$

Substituting (11) in (7) to (9), we get

$$(\nabla^2 + (\omega^2/V_1^2)\overline{\phi} = 0 \tag{12}$$

$$(\nabla^4 + \omega^2 B\nabla^2 + \omega^4 C)(\overline{\psi}, \overline{\phi}_2) = 0, \tag{13}$$

where

$$\begin{aligned} B &= \frac{q(p-2)}{\omega^2} + \frac{1}{(c_2^2+c_3^2)} + \frac{1}{c_4^2}, \\ C &= \frac{1}{(c_2^2+c_3^2)}\left(\frac{1}{c_4^2} - \frac{2q}{\omega^2}\right) \end{aligned} \tag{14}$$

and

$$V_1^2 = c_1^2 + c_3^2. \tag{15}$$

In an unbounded medium, the solution of (12) corresponds to modified longitudinal displacement wave (LD wave) propagating with velocity V_1.

The solution of (13) can be written as

$$\overline{\psi} = \overline{\psi}_1 + \overline{\psi}_2 \tag{16}$$

where $\overline{\psi}_1$ and $\overline{\psi}_2$ satisfy

$$(\nabla^2 + \delta_1^2)\overline{\psi}_1 = 0, \tag{17}$$

$$(\nabla^2 + \delta_2^2)\overline{\psi}_2 = 0. \tag{18}$$

and

$$\delta_1^2 = \lambda_1^2\omega^2, \quad \delta_2^2 = \lambda_2^2\omega^2, \tag{19}$$

$$\lambda_{1,2}^2 = [B \pm (B^2 - 4C)^{1/2}]/2. \tag{20}$$

From (8) we obtain

$$\overline{\phi}_2 = E\overline{\psi}_1 + F\overline{\psi}_2,$$

where

$$E = \left(\frac{\omega^2}{c_2^2 + c_3^2} - \delta_1^2\right)/p,\ F = \left(\frac{\omega^2}{c_2^2 + c_3^2} - \delta_2^2\right)/p.$$

Thus there are two waves propagating with velocities λ_1^{-1} and λ_2^{-1} each consisting of transverse displacement ψ and transverse microrotation ϕ_2. Following Parfitt & Eringen (1969), we call these waves modified coupled transverse displacement wave and transverse microrotational waves (i .e. CD I and CD II waves) respectively.

3. Formulation of the problem

We consider a model consisting two dissimilar isotropic homogeneous micropolar viscoelastic solid half-spaces separating at a loosely bonded plane interface. A Cartesian coordinate system (x, y, z) is chosen with the interface at $z = 0$ and the z-axis pointing into lower half-space. We consider plane waves in x–z plane with the wave front parallel to the y-axis. The complete geometry for incident, reflected and refracted waves is given in figure 1.

4. Boundary conditions

Following Murty (1976), the boundary conditions appropriate at the loosely bonded interface $z = 0$ are as follows.

(i) Continuity of the normal force stress across the interface $z = 0$.
(ii) Continuity of the shear force stress across the interface $z = 0$.

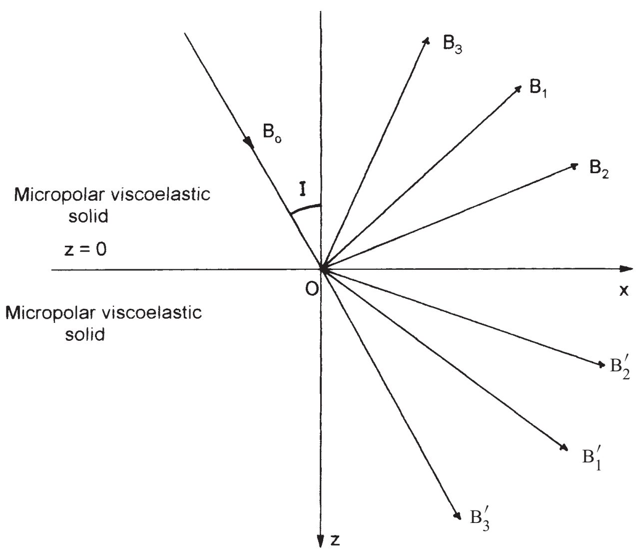

Figure 1. Geometry of the problem

(iii) Continuity of the normal displacements across the interface $z = 0$.
(iv) Continuity of the shear couple stress across the interface $z = 0$.
(v) Continuity of the microrotational component across the interface $z = 0$.
(vi) Shearing stress is proportional to the slip at the interface.

For the purpose of numerical computation, we write these boundary conditions as

$$t_{zz} = t'_{zz}, t_{zx} = t'_{zx}, u_3 = u'_3, m_{zy} = m'_{zy}, \phi_2 = \phi'_2,$$
$$t_{zx} = ik\mu\tau(u_1 - u'_1), \text{at } z = 0, \tag{21}$$

where

$$\tau = \xi/(1 - \xi)\sin \mathbf{I}, \tag{22}$$

and $\xi = 0$ corresponds to a smooth interface and $\xi = 1$ corresponds to a welded interface between half-spaces. ξ may be referred to as the bonding constant and I the angle of incidence. Symbols with primes correspond to the lower half-space ($z > 0$).

5. Reflection and transmission

We shall consider here the case when the incident wave propagates through the upper half-space ($z < 0$). An incident LD- or CD I-wave in the upper medium ($z < 0$) gives a reflected LD-wave and two reflected sets of two coupled waves (CD I and CD II), and also transmitted waves LD-, CD I- and CD II-waves in the lower medium (z > 0) as shown in figure 1.

The potential functions for incident, reflected and refracted waves {after leaving out the term, $\exp i(\omega t - kx)$} are as follows

$$\phi = B_o \exp(id\beta z) + B_1 \exp(-id\beta z), \tag{23}$$
$$\psi = B_o \exp(id\alpha_1 z) + B_2 \exp(-id\alpha_1 z) + B_3 \exp(-id\alpha_2 z), \tag{24}$$
$$\phi_2 = EB_o \exp(id\alpha_1 z) + EB_2 \exp(-id\alpha_1 z) + FB_3 \exp(-id\alpha_2 z), \tag{25}$$
$$\phi' = B'_1 \exp(id\beta' z), \tag{26}$$
$$\psi' = B'_2 \exp(id\alpha'_1 z) + B'_3 \exp(id\alpha'_2 z), \tag{27}$$
$$\phi'_2 = E' B'_2 \exp(id\alpha'_1 z) + F' B'_3 exp(id\alpha'_2 z), \tag{28}$$

where

$$d\beta = k\{(c/V_1)^2 - 1\}^{1/2}, d\alpha_1 = k\{(c\lambda_1)^2 - 1\}^{1/2}, d\alpha_2 = k\{(c\lambda_2)2 - 1\}^{1/2},$$
$$d\beta' = k\{(c/V'_1)^2 - 1\}^{1/2}, d\alpha'_1 = k\{(c\lambda'_1)^2 - 1\}^{1/2}, d\alpha'_2 = k\{(c\lambda'_2)2 - 1\}^{1/2},$$

and c is the apparent phase velocity.

For incident LD wave, $B_o = 0$ in (24) and (25) whereas for incident CD I wave, $B_o = 0$ in (23).

Making use of the potentials given by (23) to (28) in the boundary conditions given by (21), we obtain the following system of non-homogeneous equations

$$\sum_{i=1}^{6} a_{ij} Z_j = b_i, \tag{29}$$

where

$$a_{11} = -\lambda(d\beta^2 + k^2) - (2\mu + \kappa)d\beta^2, \quad a_{12} = (2\mu + \kappa)\ k\ d\alpha_1,$$
$$a_{13} = (2\mu + \kappa)\ k\ d\alpha_2, \quad a_{14} = \lambda'(d\beta'^2 + k^2) + (2\mu' + \kappa')d\beta'^2,$$
$$a_{15} = (2\mu' + \kappa')kd\alpha_1', \quad a_{16} = (2\mu' + \kappa')kd\alpha_2',$$
$$a_{21} = -(2\mu + \kappa)kd\beta, \quad a_{22} = \{\mu(k^2 - d\alpha_1^2) - \kappa(d\alpha_1^2 + E)\},$$
$$a_{23} = \{\mu(k^2 - d\alpha_2^2) - \kappa(d\alpha_2^2 + F)\}, \quad a_{24} = -(2\mu' + \kappa')kd\beta',$$
$$a_{25} = -\{\mu'(k^2 - d{\alpha_1}'^2) - \kappa'(d{\alpha_1}'^2 + E')\},$$
$$a_{26} = -\{\mu'(k^2 - d{\alpha_2}'^2) - \kappa'(d\alpha^{2'2} + F')\},$$
$$a_{31} = -d\beta, \quad a_{32} = a_{33} = k = -a_{35} = -a_{36}, \quad a_{34} = -d\beta',$$
$$a_{41} = a_{44} = 0, \quad a_{42} = -d\alpha_1\gamma E, \quad a_{43} = -d\alpha_2\gamma F,$$
$$a_{45} = -d\alpha_1'\gamma' E', \quad a_{46} = -d\alpha_2'\gamma' F', \quad a_{51} = a_{54} = 0, \quad a_{52} = E,$$
$$a_{53} = F, \quad a_{55} = -E', \quad a_{56} = -F', \quad a_{61} = -(2\mu + \kappa)kd\beta - \mu\tau k^2,$$
$$a_{62} = \{\mu(k^2 - d\alpha_1^2) - \kappa(d\alpha_1^2 + E)\} - \mu\tau kd\alpha_1,$$
$$a_{63} = \{\mu(k^2 - d\alpha_2^2) - \kappa(d\alpha_2^2 + F)\} - \mu\tau kd\alpha_2,$$
$$a_{64} = \mu\tau k^2, \quad a_{65} = -\mu\tau kd\alpha_1', \quad a_{66} = -\mu\tau kd\alpha_2'.$$

(a) *Incident LD wave*

$$b_1 = -a_{11}, \quad b_2 = a_{21}, \quad b_3 = a_{31}, \quad b_4 = -a_{41}, \quad b_5 = a_{51},$$
$$b_6 = -(2\mu + \kappa)kd\beta + \mu\tau k^2,$$

(b) *Incident CD I wave*

$$b_1 = a_{12}, \quad b_2 = -a_{22}, \quad b_3 = -a_{32}, \quad b_4 = a_{42}, \quad b_5 = -a_{52},$$
$$b_6 = -\{\mu(k^2 - d\alpha_1^2) - \kappa(d\alpha_1^2 + E)\} - \mu\tau kd\alpha_1,$$

and $Z_i (i = 1, 2, \ldots, 6)$ are the amplitude ratios for various reflected and transmitted waves.

If we neglect viscous effects, the system of equations reduce to that for the problem of reflection and transmission of plane waves at a loosely bonded interface between two dissimilar micropolar elastic solid half-spaces. The problems on loosely bonded interface discussed by Murty (1976) and Kumar & Singh (1997) may be derived as particular cases of the present problem.

6. Numerical results and discussion

Theory indicates that the amplitude ratios $|Z_i|$, $(1, 2, \ldots, 6)$ depend on the angle of incidence of the incident wave. To study in greater detail, the dependence of these ratios on properties of media together with the angle of incidence, we compute the amplitude ratios. Following Gauthier (1982), the physical constants used for aluminium–epoxy composite (micropolar elastic solid) are

$$\lambda^{*\prime} = 7.59 \times 10^{11} \text{ dyne/cm}^2, \mu^{*\prime} = 1.89 \times 10^{11} \text{ dyne/cm}^2,$$
$$\kappa^{*\prime} = 0.0149 \times 10^{11} \text{dyne/cm}^2, \rho' = 2.19 \text{ gm/cm}^3,$$
$$\gamma^{*\prime} = 0.0268 \times 10^{11} \text{dyne}, j' = 0.0196 \text{ cm}^2.$$

The physical constants for a particular model of micropolar viscoelastic solid are given as

$$\lambda' = \lambda^{*\prime}(1 + iQ_1{'}^{-1}), \qquad \mu' = \mu^{*\prime}(1 + iQ_2{'}^{-1}),$$
$$\kappa' = \kappa^{*\prime}(1 + iQ_3{'}^{-1}), \qquad \gamma' = \gamma^{*\prime}(1 + iQ_4{'}^{-1}),$$

where the quality factors $Q_i'(i = 1, 2, \ldots, 4)$ are chosen arbitrarily as

$$Q_1' = 5, \quad Q_2' = 10, \quad Q_3' = 15, \quad Q_4' = 13.$$

This numerical data is used for lower half-space.

The numerical data used for upper half-space is as follows

$$\lambda^* = 6.8 \times 10^{11} \text{ dyne/cm}^2, \qquad \mu^* = 1.63 \times 10^{11} \text{ dyne/cm}^2,$$
$$\kappa^* = 0.0134 \times 10^{11} \text{dyne/cm}^2, \qquad \rho = 2.06 \text{ gm/cm}^3,$$
$$\gamma^* = 0.0254 \times 10^{11} \text{ dyne}, \qquad j = 0.0187 \text{ cm}^2,$$
$$\lambda = \lambda^*(1 + iQ_1^{-1}), \qquad \mu = \mu^*(1 + iQ_2^{-1}),$$
$$\kappa = \kappa^*(1 + iQ_3^{-1}), \qquad \gamma = \gamma^*(1 + iQ_4^{-1}),$$

where quality factors $Q_i(i = 1, 2, \ldots, 4)$ are chosen arbitrarily as

$$Q_1 = 4, \quad Q_2 = 9, \quad Q_3 = 13, \; Q_4 = 11.$$

For the above values of relevant physical constants, the system of (29) are solved for amplitude ratios by the application of the Gauss elimination method for different angles of incidence varying from 0° to 90°. The variations of the modulus of the amplitude ratios of various reflected and transmitted waves are shown graphically with the angle of incidence of the incident LD or CDI wave for the bonding parameter $\xi = 0.0, 0.25, 0.50, 0.75, 1.0$ and for frequency $\omega^2/\omega_o^2 = 20$. The nature of dependence of amplitude ratios of different reflected and transmitted waves on the angle of incidence is, however, different for the different values of the bonding parameter.

6.1 *Incident LD wave*

Figure 2 shows the variations of amplitude ratios for reflected LD wave (without centre symbols) and transmitted LD wave (with centre symbol) with the angle of incidence for $\xi = 0.0, 0.25, 0.50, 0.75$ and 1.0. For each bonding parameter, the amplitude ratios for reflected LD wave first decrease to their minimum values and then attain their respective maxima. The amplitude ratios for transmitted wave decrease from their maxima to minima for each value of bonding parameter. The amplitude ratios for reflected LD and transmitted LD wave vary with the change in the value of bonding parameter at each angle of incidence.

Figure 3 shows the variations of amplitude ratios for reflected CD I waves (without centre symbols) and transmitted CD I waves (with centre symbols) with the angle of incidence for $\xi = 0.0, 0.25, 0.50, 0.75, 1.0$ and for frequency ratio $\omega^2/\omega_o^2 = 20$. On comparing the curves without centre symbols in figure 3, the effect of bonding parameter is observed on reflected CD I waves. Similarly, the transmitted CD I waves are also affected by change in bonding parameter.

Figure 4 shows the variations for reflected and transmitted CD II waves. The variations are found similar to those of reflected and transmitted CD I waves shown in figure 3. The reflected and transmitted CD II waves are also affected by change in bonding parameter.

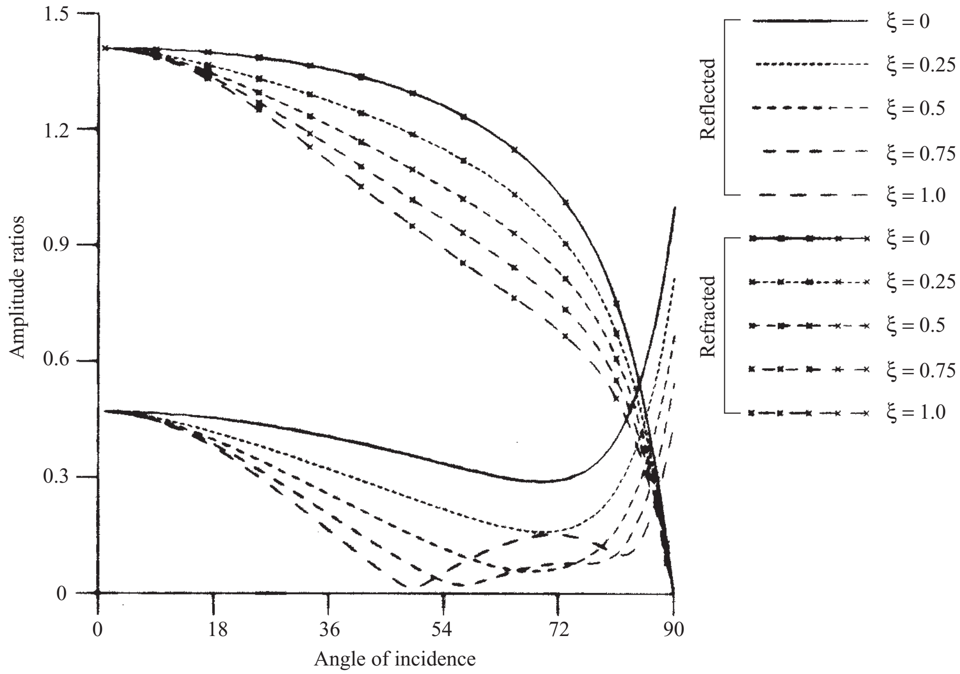

Figure 2. Variations of the amplitude ratios for reflected and refracted LD waves with the angle of incidence of incident LD wave.

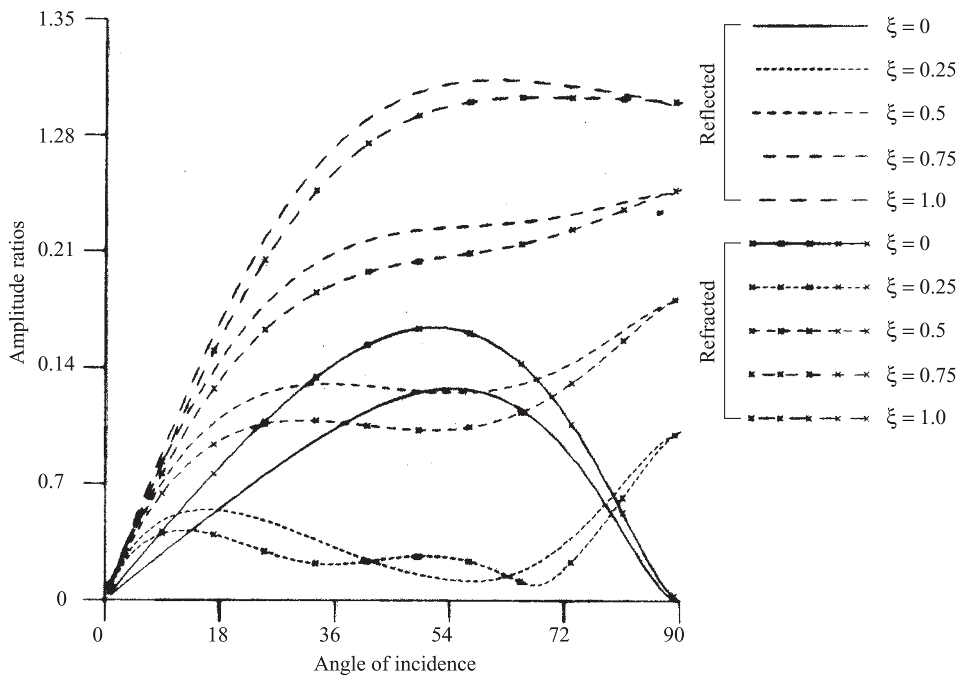

Figure 3. Variations of the amplitude ratios for reflected and refracted CD I waves with the angle of incidence of incident LD wave.

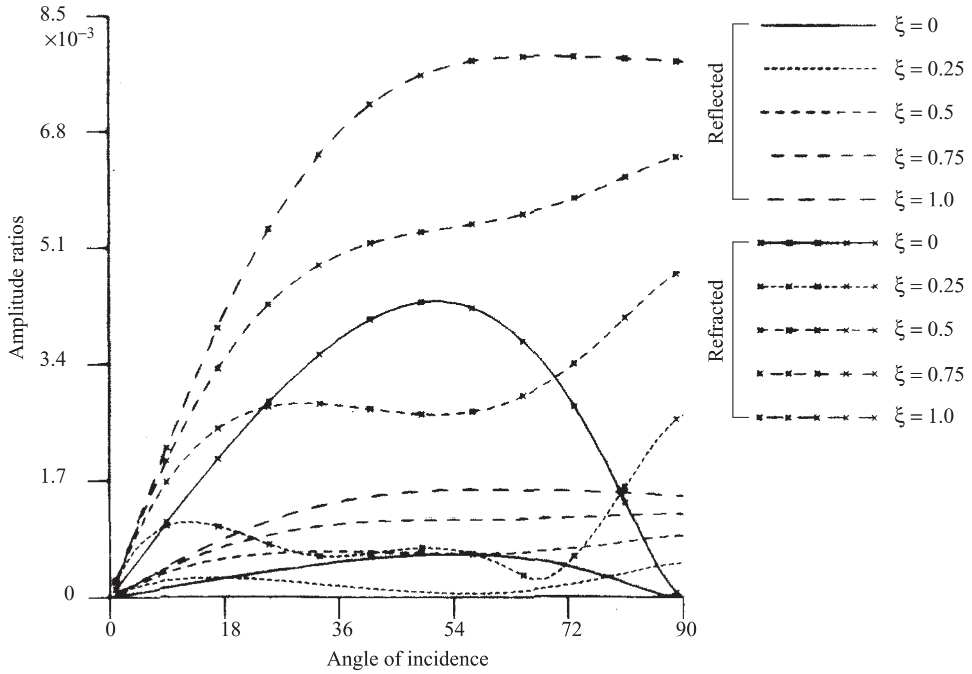

Figure 4. Variations of the amplitude ratios for reflected and refracted CD II waves with the angle of incidence of incident LD wave.

For bonding parameter $\xi = 0.5$, the amplitude ratios for reflected LD, reflected CD I, transmitted LD, transmitted CD I waves are shown in figure 5 with and without viscous effect by curves with and without centre symbols. The comparison between curves S^* and S reveals the effect of viscosity upon amplitude ratios of reflected LD wave. Similarly, the effect of viscosity is observed on reflected CD I waves, transmitted LD wave and transmitted CD I waves by comparing the curves D_1^*, D_2^*, D_3^* with the curves D_1, D_2, D_3 in figure 5. The transmitted CD I wave is least affected wave by viscosity.

For bonding parameter $\xi = 0.5$, the amplitude ratios for reflected CD II and transmitted CD II waves are shown in figure 6 with and without viscous effect by curves with and without centre symbols. The comparison between curves L^* and L reveals the effect of viscosity upon amplitude ratios of reflected CD II waves. Similarly, the effect of viscosity is observed on transmitted CD II waves by comparing the curves L_1^* with the curves L_1 in figure 6.

6.2 *Incident CD I wave*

Figures 7 to 9 show the variations of the modulus of the amplitude ratios for various reflected and transmitted waves with the angle of incidence of the incident CD I wave for the bonding parameter $\xi = 0.0, 0.25, 0.50, 0.75, 1.0$ and frequency ratio $\omega^2/\omega_o^2 = 20$. Effect of loose boundary on amplitude ratios is observed significantly.

For bonding parameter $\xi = 0.5$, the amplitude ratios for various reflected and transmitted waves are shown in figures 10 and 11 with and without viscous effect by curves with and without centre symbols. The effect of viscosity on various reflected and transmitted waves is observed.

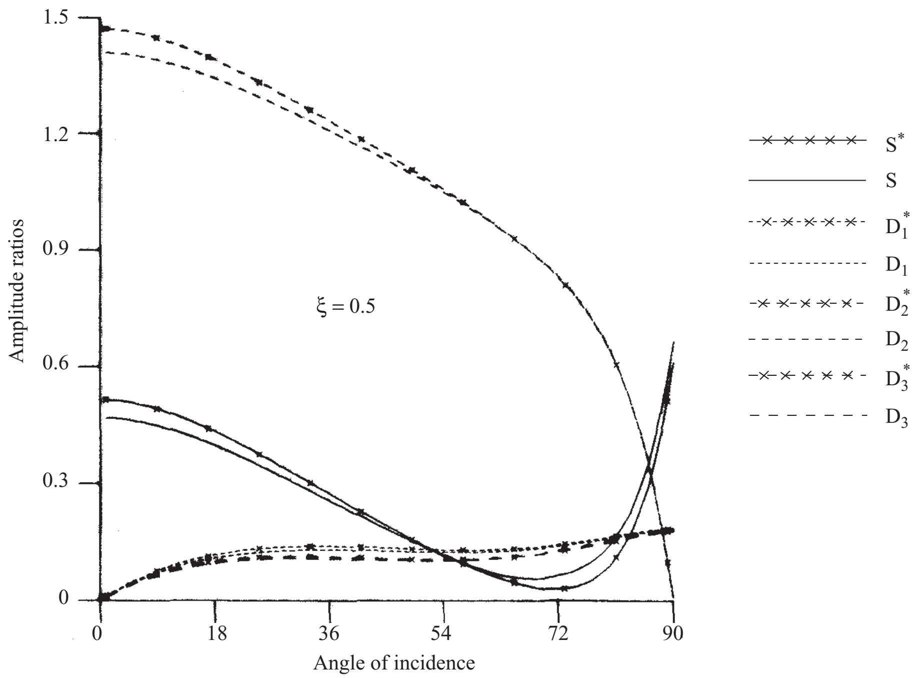

Figure 5. Variations of the amplitude ratios for reflected and refracted LD and CD I waves with the angle of incidence of incident LD wave.

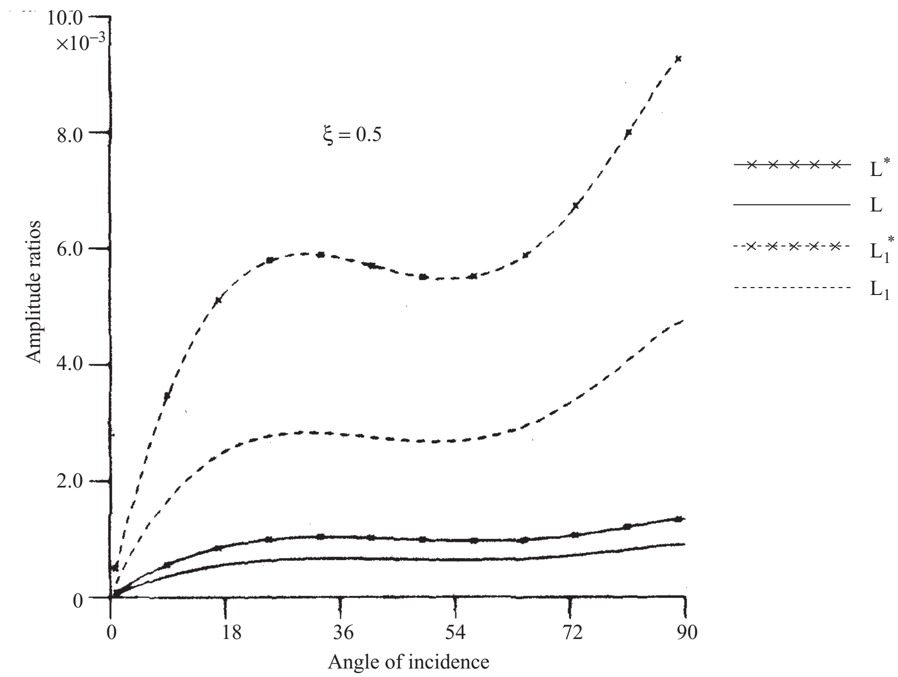

Figure 6. Variations of the amplitude ratios for reflected and refracted CD II waves with the angle of incidence of incident LD wave.

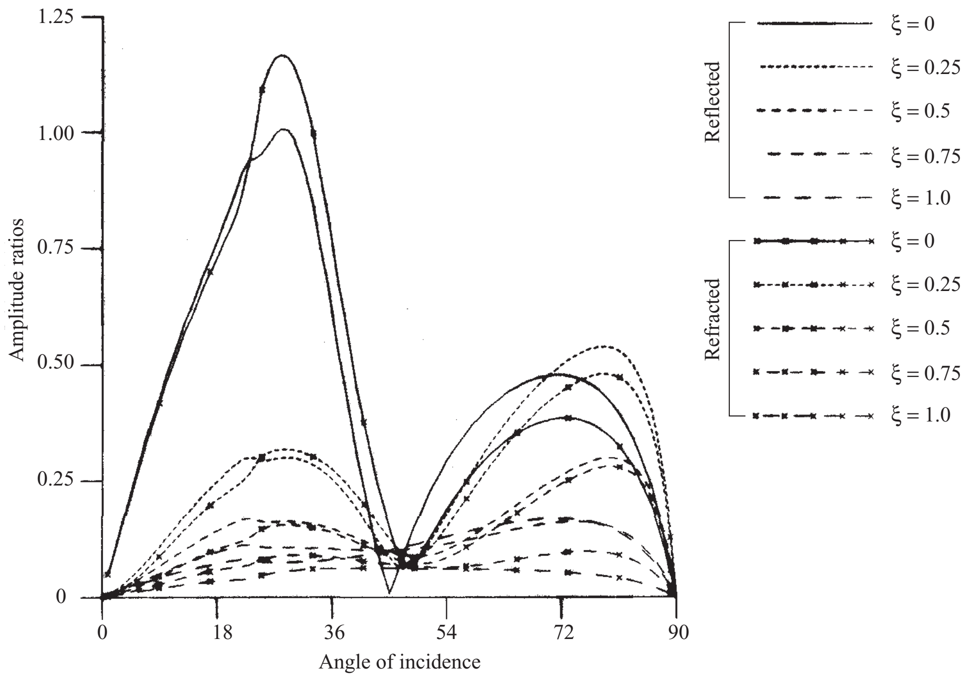

Figure 7. Variations of the amplitude ratios for reflected and refracted LD waves with the angle of incidence of incident CD I wave.

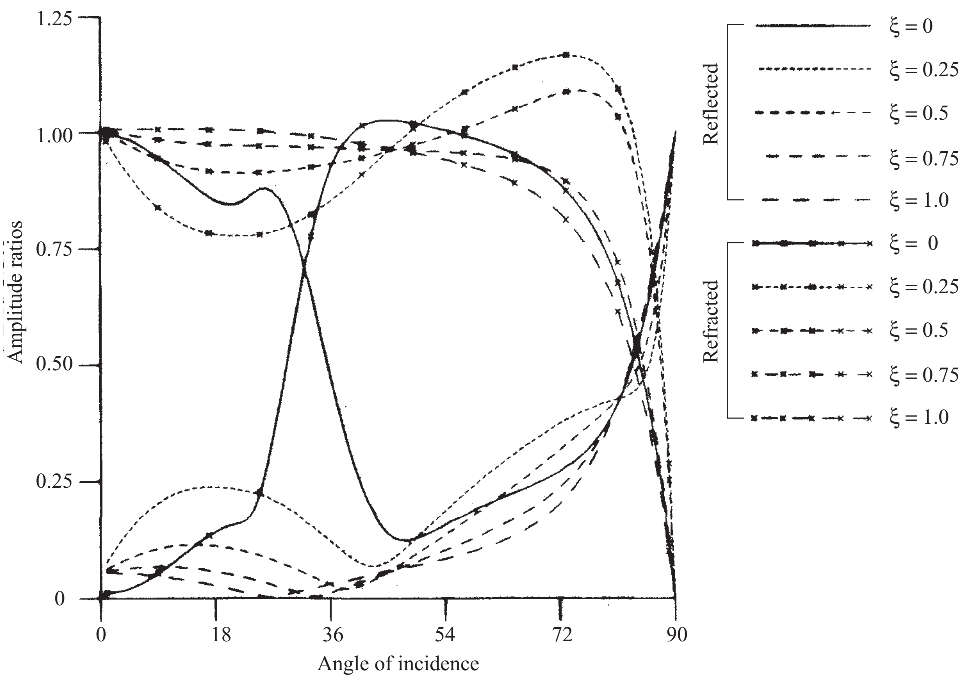

Figure 8. Variations of the amplitude ratios for reflected and refracted CD I waves with the angle of incidence of incident CD I wave.

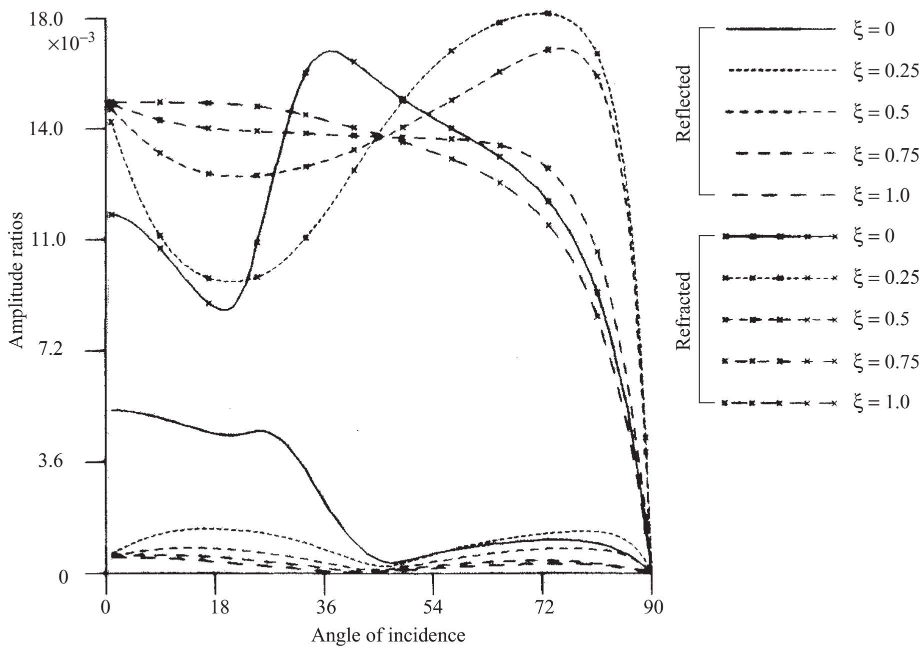

Figure 9. Variations of the amplitude ratios for reflected and refracted CD II waves with the angle of incidence of incident CD I wave.

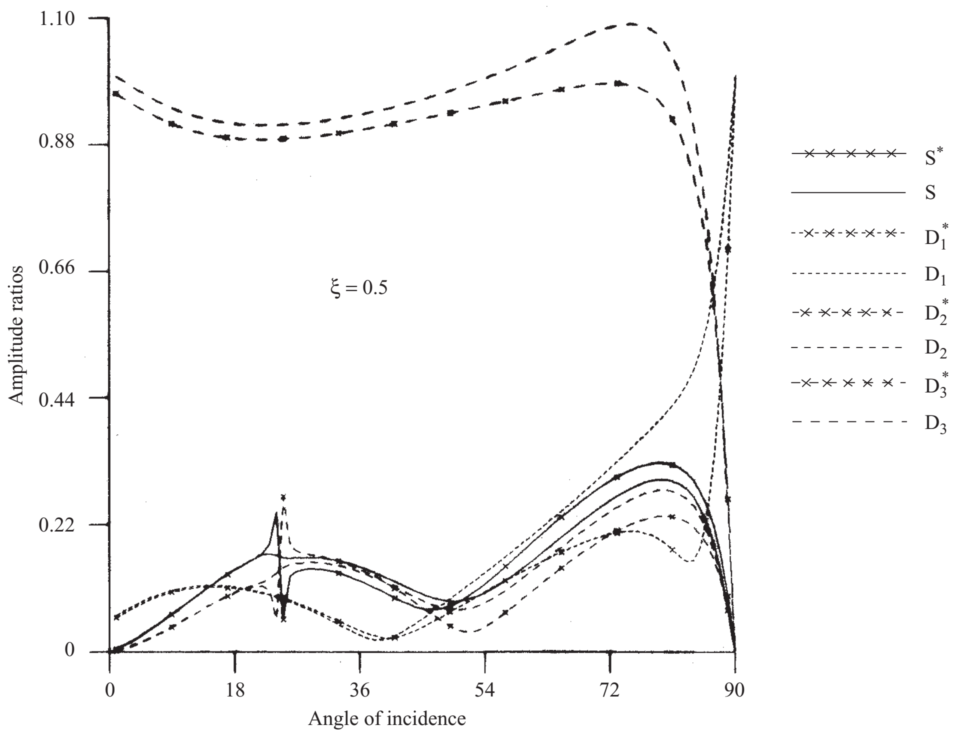

Figure 10. Variations of the amplitude ratios for reflected and refracted LD and CD I waves with the angle of incidence of incident CD I wave.

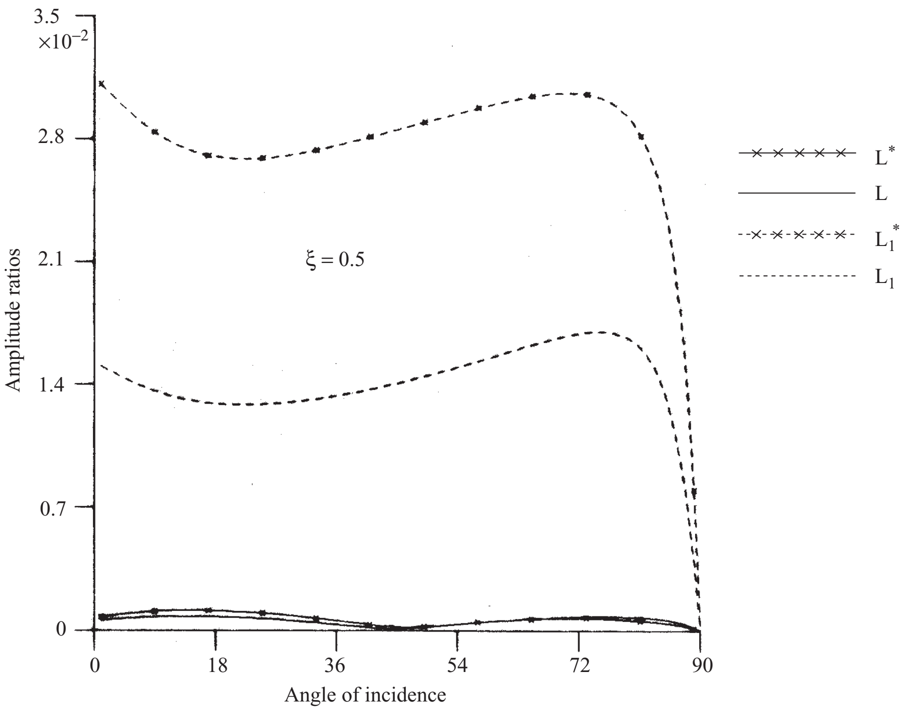

Figure 11. Variations of the amplitude ratios for reflected and refracted CD II waves with the angle of incidence and incident CD I wave.

7. Conclusions

Numerical calculations in detail are presented for the cases of LD and CD I waves incident at the loosely bonded interface of the model considered. The present numerical results agree fairly well with those obtained by Murty (1976). For the both cases of incidence, it is observed that the amplitude ratios change with the change in bonding parameter ξ. However, the rate of change of the amplitude ratios is not uniform. Therefore, the assumption of loosely bonded interface instead of welded affects reflection–refraction phenomenon more significantly. The effect of viscosity is also observed on various reflected and transmitted waves.

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