*Indian J. pure appl. Math.*, **29**(6) : 657-669, June 1998


# REFLECTION OF PLANE WAVES FROM THE FLAT BOUNDARY OF A MICROPOLAR GENERALIZED THERMOELASTIC HALF-SPACE WITH STRETCH

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(*Received* 16 *June* 1996; *after Revision* 15 *January* 1998; *Accepted* 19 *February* 1998)

In the present investigation, we have discussed the reflection of plane waves in micropolar generalized thermoelastic solid half-space with stretch. The reflection coefficients of various reflected waves with the angle of incidence for Green-Lindsay and Lord-Shulman theories have been obtained. The thermal and stretch effects are observed on various reflected waves and have been depicted graphically. A special case has been deduced.

**Key Words : Micropolar Generalized Thermoelastic Solid; Stretch; Reflection; Reflection Coefficients**

## INTRODUCTION

Theory of micropolar continua was proposed by Eringen[1] to describe the continuum behaviour of the materials possessing micro-structure. The propagation of plane waves in an infinite micropolar elastic solid have been discussed by many researchers[2-4].

Eringen[5-7] extended his work to include the effect of axial stretch during the rotation of molecules and developed the theory of micropolar elastic solid with stretch. Composite materials reinforced with chopped elastic fibres, porous media whose pores are filled up with gas or inviscid liquid, asphalt, or other elastic inclusions and 'solid-liquid' crystals etc. should be characterizable by microstretch solids[8, 9].

Lord and Shulman[10] and Green-Lindsay[11] developed generalized theory of thermoelasticity by including the thermal relaxation in time in the constitutive equations of coupled theory of thermoelasticity. These theories eliminate the paradox of infinite velocity of heat propagation. Some problems on reflection in thermoelastic solids have been discussed by Deresiewicz[12], Sinha and Sinha[13] and Sharma[14].

The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effect by Eringen[15] and Nowacki[16] and is known as micropolar coupled thermoelasticity. Dost and Tabarrok[17] presented the generalized micropolar thermoelasticity by using Green-Lindsay theory. Wave propagation in a micropolar generalized thermoelastic body with stretch has been studied by Kumar and Singh[18]. Following Lord and Shulman[10] and Green and Lindsay[11]; we study the reflection of micropolar thermoelastic waves with stretch taking into consideration the thermal relaxation in time at a solid half-space.

## BASIC EQUATIONS

We consider a homogeneous micropolar generalized thermoelastic solid with stretch that occupies the half-space. A Cartesian co-ordinate system $(x, y, z)$ is chosen with a stress-free surface $z = 0$ and $z$-axis pointing into the solid half-space. We consider plane waves in the $x$-$z$ plane with wave front parallel to the $y$-axis. Following Eringen[5], Lord and Shulman[10] and Green Lindsay[11], the constitutive relations and field equations can be written as:

$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \varepsilon_{ijr}\omega_r) - \nu(\theta + t_1\dot{\theta})\delta_{ij} \quad \text{... (1)}$$

$$m_{ij} = \beta_0 \varepsilon_{kij}\phi^*_{,k} + \alpha\omega_{r,r}\delta_{ij} + \beta\omega_{i,j} + \gamma\omega_{j,i}, \quad \text{... (2)}$$

$$\beta_j = \alpha_0 \phi^*_{,j} + \frac{\beta_0}{3}\varepsilon_{kij}\omega_{k,i}, \quad \text{... (3)}$$

$$(C_1^2 + C_3^2)\,\nabla(\nabla\cdot \boldsymbol{u}) - (C_2^2 + C_3^2)\,\nabla\times(\nabla\times\boldsymbol{u}) + C_3^2\,\nabla\times w - \bar{\nu}\,\mathrm{grad}(\theta + t_1\dot{\theta}) = \frac{\partial^2 \boldsymbol{u}}{\partial t^2}, \quad \text{... (4)}$$

$$(C_4^2 + C_5^2)\nabla(\nabla\cdot w) - C_4^2\,\nabla\times(\nabla\times w) + \omega_0^2\,\nabla\times\boldsymbol{u} - 2\,\omega_0^2\, w = \frac{\partial^2 w}{\partial t^2}, \quad \text{... (5)}$$

$$K^*\,\nabla^2\theta = \rho C^*(\dot{\theta} + t_0\ddot{\theta}) + \nu\theta_0[\dot{u}_{i,i} + \delta_{1k}t_0\ddot{u}_{i,i}], \quad \text{... (6)}$$

and

$$\alpha_0\nabla^2\phi^* - \eta_0\phi^* = \frac{\rho j}{2}\frac{\partial^2\phi^*}{\partial t^2}, \quad \text{... (7)}$$

where

$$c_1^2 = (\lambda + 2\mu)/\rho,\ c_2^2 = \mu/\rho,\ c_3^2 = K/\rho,\ c_4^2 = \gamma/\rho j,$$

$$c_5^2 = (\alpha + \beta)/\rho j,\ \omega_0^2 = K/\rho j,\ \nu = (3\lambda + 2\mu + K)\alpha_t,\ \bar{\nu} = \nu/\rho,$$

where $\lambda, \mu, K, \alpha, \beta, \gamma, \alpha_0, \beta_0, \eta_0$ are material constants, $\rho$ the density, $j$ the microrotational inertia, $K^*$ the coefficient of thermal conductivity, $\alpha_t$ the coefficient of linear expansion, $C^*$ the specific heat at constant strain $\theta_0$ the initial uniform temperature, $t_0, t_1$, are the thermal relaxation times, $\delta_{ij}$ the kronecker deltat, $\boldsymbol{u}$ the displacement vector, $w$ the microrotation vector and $\phi^*$ the scalar microstretch. The superposed dot denotes the derivative with respect to time.

For the L-S (Lord-Shulman) theory $t_1 = 0, \delta_{1k} = 1$ and for *G-L* (Green-Lindsay) theory $t_1 > 0$ and $\delta_{1k} = 0$ ($k$ = 1 for L-S and 2 for G-L theory). The thermal relaxations $t_0$ and $t_1$ satisfy the inequality $t_1 \geq t_0 \geq 0$ for the G-L theory only.

By Helmoholtz representation of a vector, we can write

$$\boldsymbol{u} = \nabla q + \nabla\times\boldsymbol{U}, \qquad \nabla\cdot\boldsymbol{U} = 0, \quad \text{... (8)}$$

$$w = \nabla\zeta + \nabla\times\Phi, \qquad \nabla\cdot F = 0, \quad \text{... (9)}$$

Since we are discussing a two-dimensional problem in the $xz$-plane, we have

$$\boldsymbol{u} = (u_1, 0, u_3), \quad \boldsymbol{f} = (0, \phi_2, 0) \qquad \ldots (10)$$

Eliminating $\theta$ from eq. (4) and (6) and making use of eqs. (8) and (10), we get

$$\nabla^4 q - \left[\frac{C^*}{\bar{K}^*}\left(1 + t_0\frac{\partial}{\partial t}\right) + \varepsilon\left(1 + t_1\frac{\partial}{\partial t}\right)\left(1 + \delta_{1k} t_0\frac{\partial}{\partial t}\right) + \frac{1}{V_1^2}\frac{\partial}{\partial t}\right]\frac{\partial}{\partial t}\nabla^2 q$$

$$+ \frac{C^*}{\bar{K}^*}\frac{1}{V_1^2}\left(1 + t_0\frac{\partial}{\partial t}\right)\frac{\partial^3 q}{\partial t^3} = 0. \qquad \ldots (11)$$

Similarly, eqs. (4) and (5) with the help of eqs. (8)-(10) by eliminating $\phi_2\,[= -(\phi)_y]$ yield

$$\nabla^4 U_y - \left[\left(\frac{1}{b^2} + \frac{1}{c_4^2}\right)\frac{\partial^2}{\partial t^2} - \frac{\omega_0^2}{C_4^2}\left(\frac{C_3^2}{b^2} - 2\right)\right]\nabla^2 U_y + \frac{1}{b^2 C_4^2}\left(2\omega_0^2 + \frac{\partial^2}{\partial t^2}\right)\frac{\partial^2 U_y}{\partial t^2} = 0,$$

$$\ldots (12)$$

where

$$\bar{K}^* = \frac{K^*}{\rho}, \quad V_1^2 = C_1^2 + C_3^2, \quad b^2 = C_2^2 + C_3^2, \; U_y = (-U)_y \;\text{ and }\; \varepsilon = \frac{\nu^{-2}\theta_0}{C^* V_1^2}.$$

## Boundary Conditions

The appropriate boundary conditions are

$$t_{zz} = t_{zx} = m_{zy} = \frac{\partial\theta}{\partial z} = \beta_z = 0, \qquad \text{at} \quad z = 0. \qquad \ldots (13)$$

## Formulation of the Problem and its Solution

When we consider the propagation of a train of plane waves in the $xz$-plane which makes an angle $I$ with the normal to the boundary, $I$ is known as the angle of incidence. For an incident longitudinal displacement wave $c = V_1$ cosec $I$, for an incident coupled transverse and microrotational waves, $c = V_3\,(= \lambda_4^{-1})$ cosec $I$ and for an incident longitudinal microstretch wave $c = V^*$ cosec $I$, where $V_1$, $V_3$ and $V^* = \left(C_6^2 \big/ \left(1 - \frac{V_0^2 C_6^2}{\omega^2}\right)\right)^{1/2}$ are velocities of longitudinal displacement wave, coupled waves and longitudinal microstretch waves respectively and $c$ is the apparent phase velocity on the surface. The complete geometry of the problem has been shown in Fig. 1.

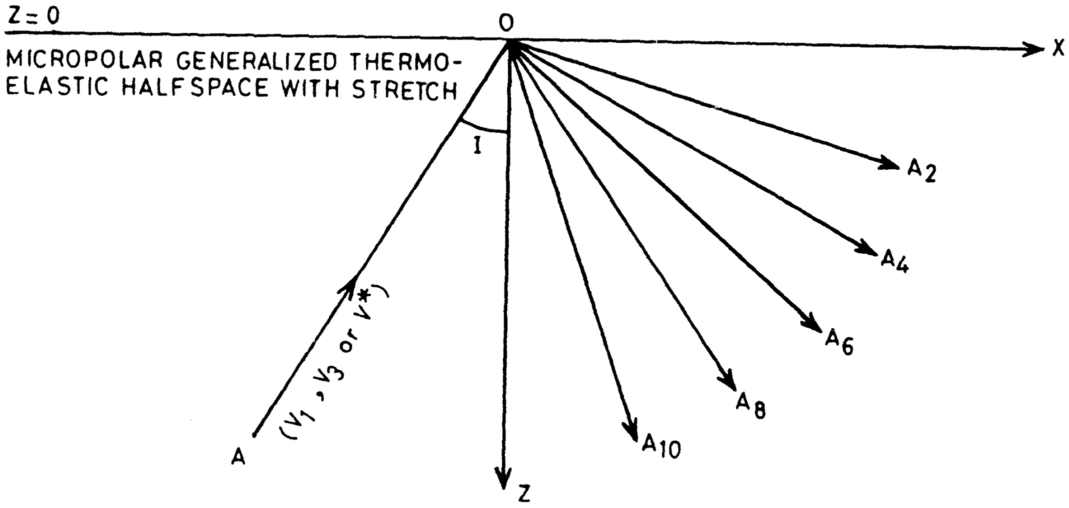



FIG. 1. Geometry of the problem

Solving eqs. (11), (12) and (7), we obtain the appropriate potentials as

$$q=[A_1 e^{\lambda_1 z}+A_2 e^{-\lambda_1 z}+A_3 e^{\lambda_2 z}+A_3 e^{\lambda_2 z}+A_4 e^{-\lambda_2 z}]\, e^{ik(ct-x)}, \quad \dots (14)$$

$$\theta=\frac{1}{\gamma_0}[b_1A_1 e^{\lambda_1 z}+b_1A_2e^{-\lambda_1 z}+b_2A_3e^{\lambda_2 z}+b_2A_4 e^{-\lambda_2 z}]\, e^{ik(ct-x)}, \quad \dots (15)$$

$$U_y=[A_5 e^{\lambda_3 z}+A_6 e^{-\lambda_3 z}+A_7 e^{\lambda_4 z}+A_8-\lambda_4 z]\, e^{ik(ct-x)}, \quad \dots (16)$$

$$\omega_2=[b_3A_5 e^{\lambda_3 z}+b_3A_6 e^{-\lambda_3 z}+b_4A_7e^{\lambda_4 z}+b_4A_8 e^{-\lambda_4 z}]\, e^{ik(ct-x)} \quad \dots (17)$$

and

$$\phi^* = G[A_9 e^{\lambda_6 \dot{z}}+A_{10} e^{-\lambda_6 z}]\, e^{ik(ct-x)}, \quad \dots (18)$$

where

$$\lambda_1=\left\{\frac{1}{2}\sqrt{A^2-4B}-A)\right\}^{1/2},\quad \lambda_2=\left\{-\frac{1}{2}(\sqrt{A^2-4B}+A)\right\}^{1/2}$$

$$\lambda_3=\left\{\frac{1}{2}\sqrt{C'^2-4D'}-C')\right\}^{1/2},\quad \lambda_4=\left\{-\frac{1}{2}(\sqrt{C'^2-4D'}+C')\right\}^{1/2}$$

$$\lambda_6=\left(\frac{\omega^2}{C_6^2}-V_0^2-k^2\right)^{1/2} \quad \dots (19)$$

and

$$A = k^2\left(\frac{C^2}{V_1^2}2\right) - \frac{C^*}{\overline{K}^*}k[(i - t_0\,kc) + \varepsilon\,(i - t_1\,kc)\,(1 + ikc\,t_0\,\delta_{1k})],$$

$$B = k^4\left(1 - \frac{C^2}{V_1^2}\right) + cK^3\frac{C^*}{\overline{K}^*}\left[(i - t_0kc) + \varepsilon(i - t_1\,kc)\,(1 + ikct_0\,\delta_{1k}) - \frac{C^2}{V_1^2}(i - t_0\,kc)\right],$$

$$C' = k^2\left(\frac{c^2}{b^2} + \frac{c^2}{c_4^2} - 2\right) + \frac{\omega_0^2}{C_4^2}\left(\frac{C_3^2}{b^2} - 2\right)$$

and

$$D' = k^4\left(1 - \frac{c^2}{b^2} - \frac{c^2}{C_4^2} - \frac{c^4}{b^2\,c_4^2}\right) - k^2\frac{\omega_0^2}{c_4^2}\left(\frac{c_3^2}{b^2} + \frac{2c^2}{b^2} - 2\right).$$

$G$ is a constant with dimension $L^2$

and

$$b_i = k^2\,(c^2 - V_1^2) + \lambda_i^2\,v_i^2,\ (i = 1,\ 2);\ b_i = k^2\left(\frac{b^2}{c_3^2} - \frac{c^2}{c_3^2}\right) - \lambda_i^2\left(\frac{b^2}{C_3^2}\right), \qquad (i = 3,\ 4)$$

$$\overline{\gamma}_0 = \overline{\nu}\,(1 + i\omega t_1)\,;$$

(*a*) for incident and longitudinal displacement waves,

$$A_1 = A_5 = A_7 = A_9 = 0\,; \qquad \ldots (20)$$

(*b*) for incident coupled transverse and microrotational waves

$$A_1 = A_3 = A_5 = A_9 = 0\,; \qquad \ldots (21)$$

and

(*c*) for incident longitudinal microstretch waves

$$A_1 = A_3 = A_5 = A_7 = 0. \qquad \ldots (22)$$

Making use of the potentials given by eqs. (14)-(18) in the boundary conditions (13) and with the help of eqs. (1)-(3) and (8)-(9), we obtain a system of five non-homogeneous equations which can be written as

$$\sum_{i=1}^{5} a_{ij}^*\,z_j = b_1^*, \qquad (i = 1,\ 2,\ \ldots,\ 4), \qquad \ldots (23)$$

where

$$a_{1i}^* = (\lambda + 2\mu + K)\lambda_i^2 - \lambda K^2 - \rho b_i, \qquad (i = 1,\ 2)$$

$$a_{1i} = -ik(2\mu + K)\lambda_i, \qquad (i = 3,\ 4)$$

$$a_{15}^* = 0 = a_{25}^* = a_{31}^* = a_{32}^* = a_{51}^* = a_{52}^* = a_{43}^* = a_{44}^* = a_{45}^*,$$

$$a_{2i}^* = ik(2\mu + K)\lambda_i \quad (i = 1,\ 2); \qquad a_{2i}^* = (\mu + K)\lambda_i^2 - Kb_i + \mu k^2 \qquad (i = 3,\ 4),$$

$$a_{3i}^* = \lambda_i b_i, \qquad (i = 3,\ 4); \qquad a_{35}^* = \frac{-ik\beta_0 G}{\gamma}; \qquad a_{5i}^* = ik\beta_0 b_i, \qquad (i = 3,\ 4)$$

$$a_{4i} = \lambda_i b_i, \qquad (i = 1,\ 2); \text{ and } \quad a_{55}^* = 3\alpha_0\lambda_6 G\ ;$$

($a$) for incident longitudinal displacement waves

$$b_1^* = -a_{12}^*, \quad b_2^* = a_{22}, \quad b_3^* = a_{32}^*, \quad b_4^* = a_{42}^*, \quad b_5^* = a_{52}^*\ ; \qquad \text{... (24)}$$

($b$) for incident coupled transverse and microrotational waves,

$$b_1^* = a_{14}^*, \quad b_2^* = -a_{24}, \quad b_3^* = a_{34}^*, \quad b_4^* = a_{44}^*, \quad b_5^* = -a_{54}^*\ ; \qquad \text{... (25)}$$

($c$) for incident longitudinal microstretch waves

$$b_1^* = a_{15}^*, \quad b_2^* = a_{25}, \quad b_3^* = -a_{35}^*, \quad b_4^* = a_{45}^*, \quad b_5^* = a_{55}^*\ ; \qquad \text{... (26)}$$

and

$$Z_1 = \frac{A_2}{A}, \quad Z_2 = \frac{A_4}{A}, \quad Z_3 = \frac{A_6}{A}, \quad Z_4 = \frac{A_8}{A}, \quad Z_5 = \frac{A_{10}}{A} \qquad \text{... (27)}$$

are the amplitude ratios of thermal wave, longitudinal displacement wave, two coupled waves and a longitudinal microstretch wave respectively, where

$$A = \begin{cases} A_3, & \text{for incident longitudinal displacement wave,} \\ A_7, & \text{for incident coupled wave,} \\ A_9, & \text{for incident longitudinal microstrech wave} \end{cases}$$

*Particular Cases*

($i$) For L-S theory ($t_1 = 0, \delta_{1k} = 1$), the expressions for $\lambda_1$ and $\lambda_2$ in the system of eq. (23) will be

$$\lambda_1 = \left\{ \frac{1}{2} [\sqrt{A^{*2} - 4B^*} - A^*] \right\}^{1/2}; \quad \lambda_2 = \left\{ -\frac{1}{2} [\sqrt{A^{*2} - 4B^*} + A^*] \right\}^{1/2}, \qquad \text{... (28)}$$

where

$$A^* = k^2 \left( \frac{c^2}{v_1^2} - 2 \right) - \frac{C^*}{\overline{K}^*} kc\,(1 + \varepsilon)\,(i - t_0\, kc)$$

and

$$B^* = k^4 \left( 1 - \frac{c^2}{v_1^2} \right) + ck^3 \frac{C^*}{\overline{K}^*} (i - t_0\, kc) \left( 1 + \varepsilon - \frac{c^2}{v_1^2} \right).$$

(*ii*) For G-L theory ($t_1 > 0, \delta_{1k} = 0$), the expressions for $\lambda_1$ and $\lambda_2$ in the system of eq. (23) will become

$$\lambda_1 = \left\{ \frac{1}{2} [\sqrt{A'^2 - 4B'} - A'] \right\}^{1/2}; \quad \lambda_2 = \left\{ -\frac{1}{2} [\sqrt{A'^2 - 4B'} + A'] \right\}^{1/2}, \qquad \dots (29)$$

where

$$A' = k^2 \left( \frac{c^2}{v_1^2} - 2 \right) - kc \frac{C^*}{K^*} \{(i - t_0 kc) + \varepsilon(i - t_1 kc)\}$$

and

$$B' = k^4 \left( 1 - \frac{c^2}{v_1^2} \right) + ck^3 \frac{C^*}{K^*} \left\{ (i - t_0 kc) \left( 1 - \frac{c^2}{v_1^2} \right) + \varepsilon(i - t_1 kc) \right\}.$$

## SPECIAL CASE

If we neglect both stretch and thermal effects, the expression for $\lambda_1$, $\lambda_2$ and $\lambda_6$ in the system of eq. (23) will be

$$\lambda_1 = \left\{ \frac{1}{2} [\sqrt{\bar{A}^2 - 4\bar{B}} - \bar{A}] \right\}^{1/2}; \lambda_3 = \left\{ \left( -\frac{1}{2} \right) [\sqrt{\bar{A}^2 - 4\bar{B}} + \bar{A}] \right\}^{1/2}; \lambda_6 = 0$$

where

$$\bar{A} = k^2 \left( \frac{c^2}{v_1^2} - 2 \right), \bar{B} = k^4 \left( 1 - \frac{c^2}{v_1^2} \right).$$

In this case, our results reduce to those obtained by Parfitt and Eringen[4].

## THERMAL RELAXATION IN TIME

Chester[19] argued that since $t_0$ is the time needed to establish a steady resistive flow, the rate $1/t_0$ must be connected with thermal resistance and $t_0$ must be proportional to the thermal conductivity $K^*$. Also, the exact relation between $t_0$ and $K^*$ can be found once the second sound speed in a solid has been established. He has pointed out that there is a critical lower frequency below which thermal waves will not propagate and also there is an upper frequency limit above which the concept of temperature becomes hazy and thermal waves will not exist. Lord and Shulman[10] regarded it as relaxation parameter and used the non-dimensional form of the relaxation time as $\alpha\rho C^* t_0/K^*$, $\alpha[= (\lambda + 2\mu)/\rho)^{1/2}]$ is the velocity of longitudinal waves. Nayfesh and Nasser[20] took $t_0 = 3K^*/\rho C^* \alpha^2$. According to the experiment of Peierls[21] its order should be $10^{-13}$. Lord[22] took $t_0 = 10^{-13}$. Green and Lindsay[11] considered $t_0$ and $t_1$ as relaxation times.

Since we are dealing with micropolar generalized thermoelastic solid with stretch, we consider $t_0 = 3K^*/\rho C^* V_1^2$ and $t_1$ of the same order as $t_0$. Jefferys[23] presented his results for reflections from free surface in terms of reflection coefficients. We have followed Jefferys line of calculation for our results.

## NUMERICAL ANALYSIS

We take the case of aluminium-epoxy composite subject to thermal disturbances for our calculations. The physical constants used by us are :

$$\rho = 2.19 \text{ gm/cm}^3, \quad j = 0.196 \text{ cm}^2, \quad \lambda = 7.59 \times 10^{11} \text{ dyne/cm}^2,$$

$$\mu = 1.89 \times 10^{11} \text{ dyne/cm}^2, \quad K = 0.0149 \times 10^{11} \text{ dyne/cm}^2, \quad \varepsilon = 0.073,$$

$$\gamma = 0.268 \times 10^{11} \text{ dyne}, \quad C^* = 0.23 \text{ cal/C}^{\circ}, \quad K^* = 0.6 \times 10^{-2} \text{ cal/cm sec } ^{\circ}\text{C},$$

$$t_0 = 3K^*/\rho C^* V_1^2 = 6.131 \times 10^{-3}, \quad t_1 = 8.765 \times 10^{-3}, \quad \omega^2/\omega_0^2 = 10.$$

For the above values of relevant physical constants, the system of eq. (23) in reduced form for L-S theory, G-L theory and micropolar elastic case are solved for reflection coefficients by using Gauss Elimination method for different angles of incidence varying from $0^{\circ}$ to $90^{\circ}$. The variations of reflection coefficients with the angle of incidence have been shown graphically in Figs. 2 to 16 for the incident longitudinal displacement wave (LD wave), coupled transverse and microrotational waves (coupled wave) and longitudinal microstretch waves (LMS-wave).

In all cases LS(1) and GL(1) correspond to reflection coefficients in a free micropolar generalized thermoelastic half-space for Lord-Shulman and Green-Lindsay cases respectively. Similarly, LS(2) and GL(2) correspond to reflection coefficients in a free micropolar generalized thermoelastic half-space with stretch for Lord-Shulman and Green-Lindsay cases respectively. Also F corresponds to reflection coefficients in a free micropolar elastic half-space.

*Case (i) : Incident Longitudinal Displacement Wave*

The variations of the reflection coefficients $|z_i|$. ($i$ = 1, 2, .., 5) with the angle of incidence $I$ of the incident LD-waves starting from $I = 0^{o}$ (normal incidence) to $\theta_0 = 90\ ^{\circ}$C are depicted in Figs. 2 to 6. The comparisons between LS(1), GL(1), LS(2), GL(2) and F reveals that the thermal and stretch effects play an important role in reflection phenomenon. In Fig. 2, the reflection coefficients $|z_1|$ for GL(2) has been multiplied by $10^{-2}$ to its original values. The reflection coefficients $|z_2|$ for GL(2) has been multiplied by $10^{-1}$ to its original values in Fig. 3 and the reflection coefficients $|z_3|$ for LS(1), LS(2) and GL(2) have been multiplied by $10^{-1}$, $10^{-1}$ and $10^{-2}$ respectively in Fig. 4. In Fig. 5, the reflection coefficients $|z_4|$ for LS(1), LS(2) and GL(2) have been multiplied by $10^{-1}$, $10^{-1}$ and $10^{-2}$ respectively to their original values. The reflection coefficients $|z_5|$ for GL(2) has been multiplied by $10^{-1}$ and has been shown in Fig. 6. It is observed that the variations of reflection coefficients $|z_i|$, ($i$ = 1, 2, ..., 5) for LS($i$) and GL($i$), ($i$ = 1, 2) are quite different for different values of angle of incidence $I$. It is also noticed that for incident longitudinal displacement wave, the stretch effect on reflected thermal wave. ($|z_1|$) and reflected coupled wave ($|z_3| \cdot |z_4|$) is significance for GL-theory as compared to that in LS-theory, whereas it is observed as the maximum of LS-theory in case of reflected LD-waves. In Fig. 6, the amplitude ratios $|z_5|$ corresponds to reflected longitudinal microstretch waves which have different variations for LS-theory and GL-theory.

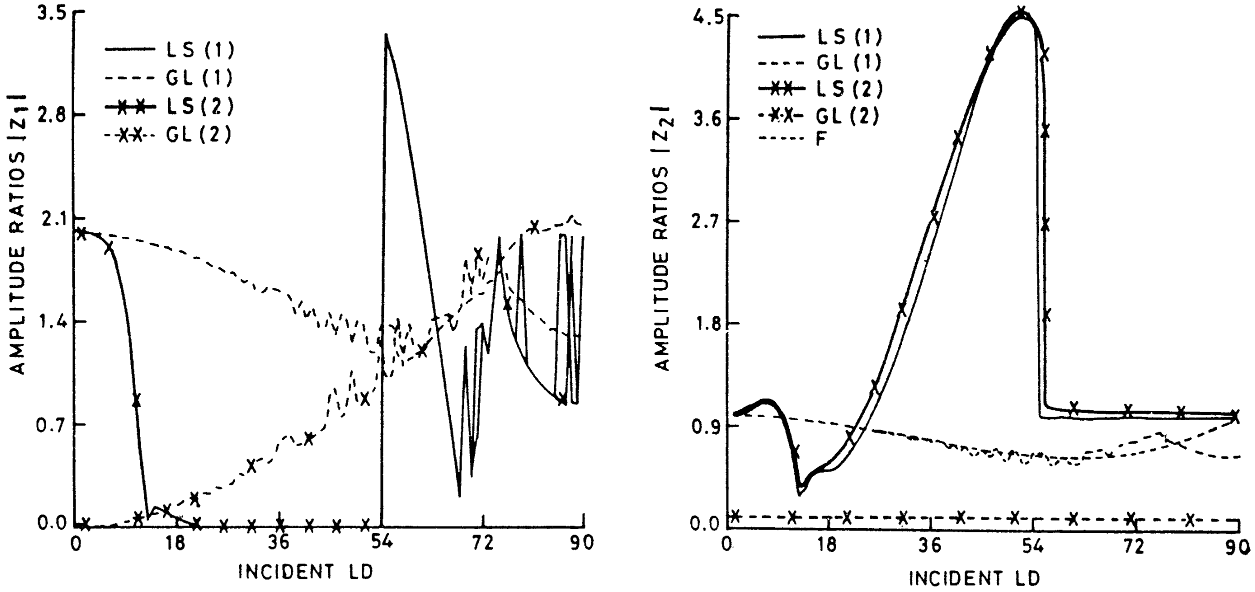



FIG. 2&3. Variations of the amplitude ratios with incident angle of L D-wave

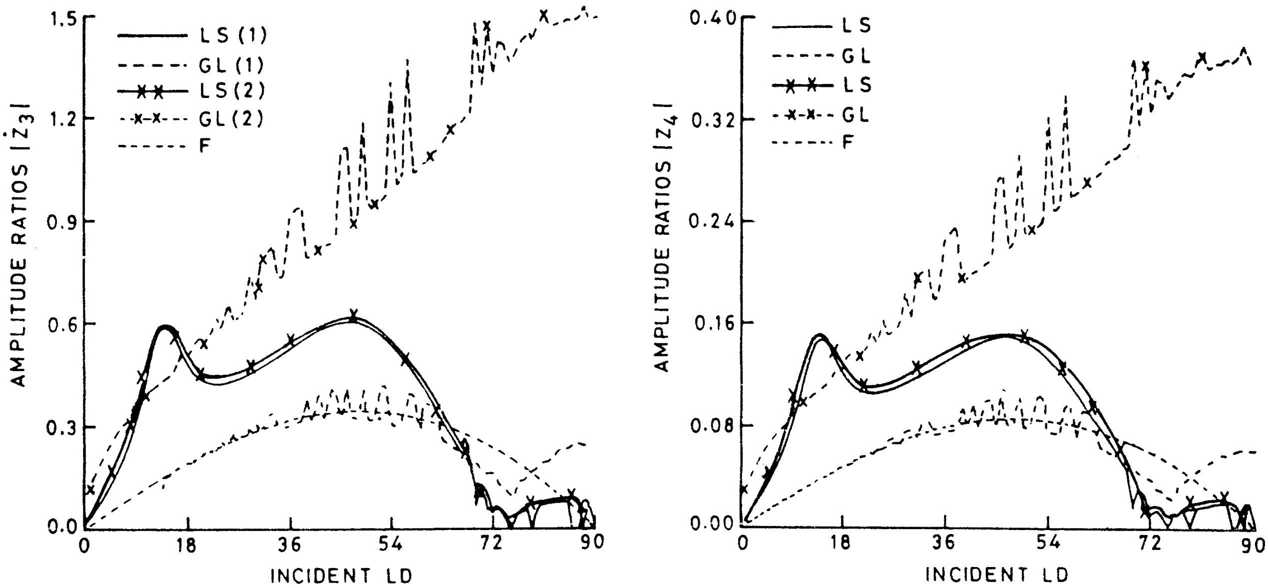



FIG. 4&5. Variations of the amplitude ratios with incident angle of L D-wave

*Case (ii) : Incident Coupled Transverse and Microrotational Wave*

The variations of reflection coefficients $|z_i|$, ($i$ = 1, 2, ..., 5) with the angle of incidence $I$ have been shown graphically in Figs. 7 to 11. Likewise the case of incident LD-wave, the stretch and thermal effects are observed on all reflection coefficients. The reflection coefficient $|z_1|$ for LS(2) and GL(2) has been multiplied by $10^{-3}$ and $10^{-2}$ to their original value and has been shown in Fig. 7. The reflection coefficients $|z_2|$ for reflection LD-wave have been shown in Fig. 8, by multiplying its original values by $10^{-2}$ and $10^{5}$ for LS(2) and GL(2) cases respectively. The reflection

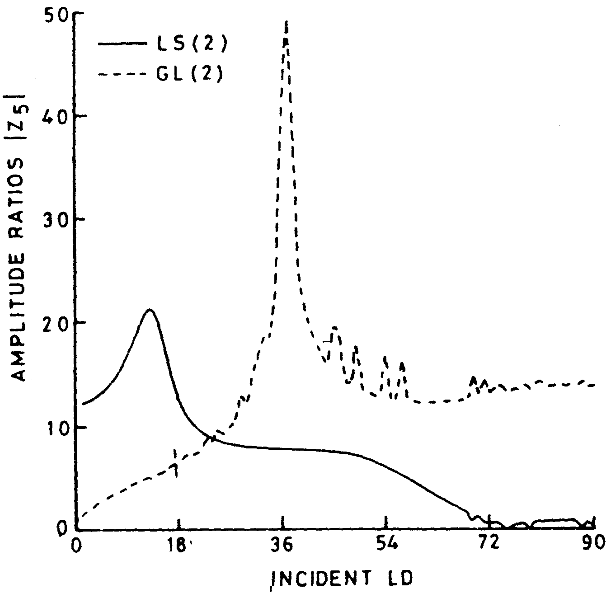



FIG. 6. Variations of the amplitude ratios with incident angle of L D-wave

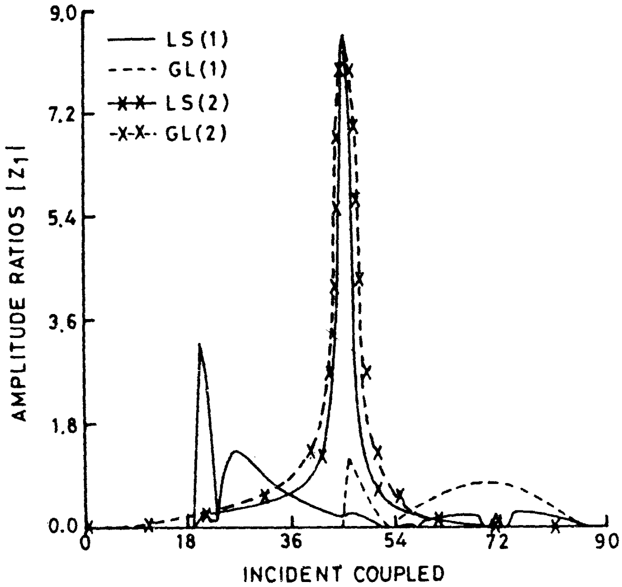
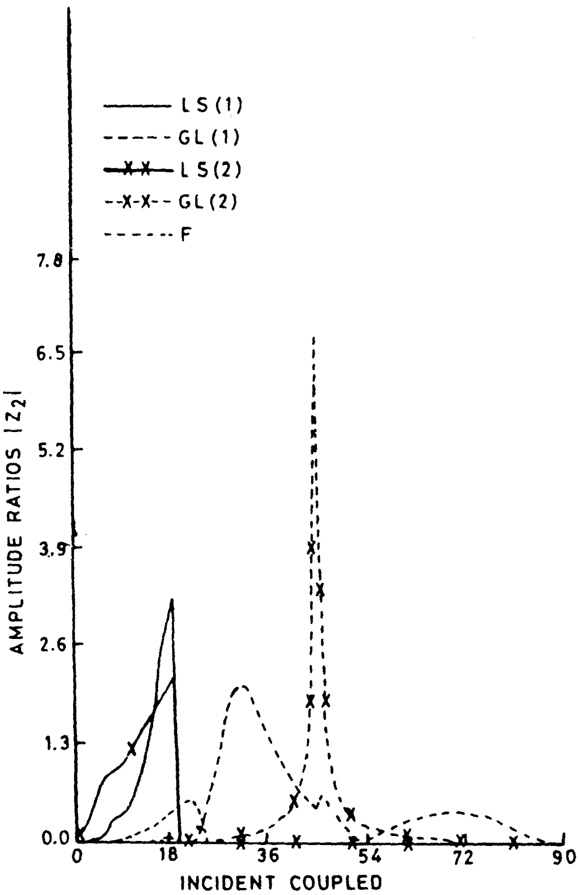



FIG. 7. Variations of the amplitude ratios with coupled-wave



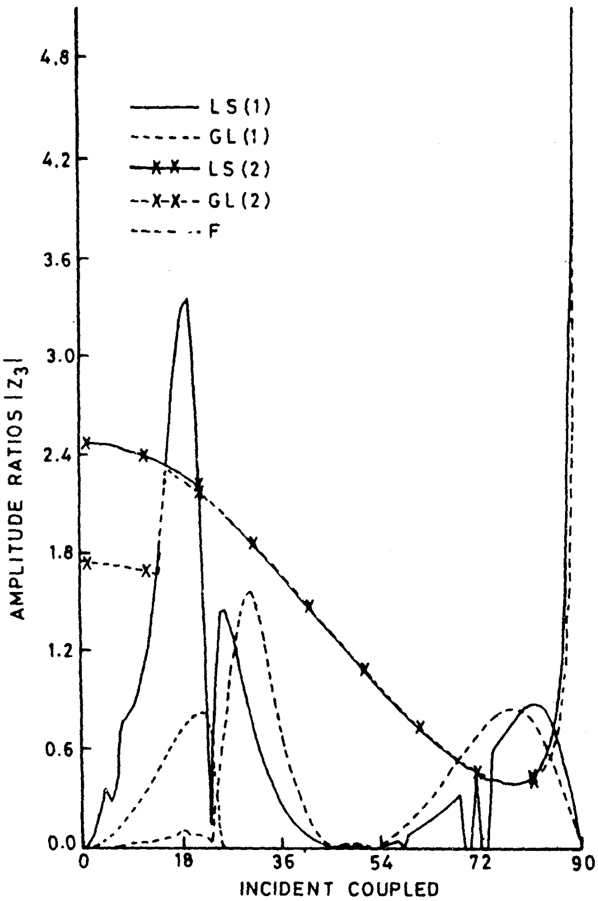




FIG. 8&9. Variations of the amplitude ratios with incident angle of coupled-wave

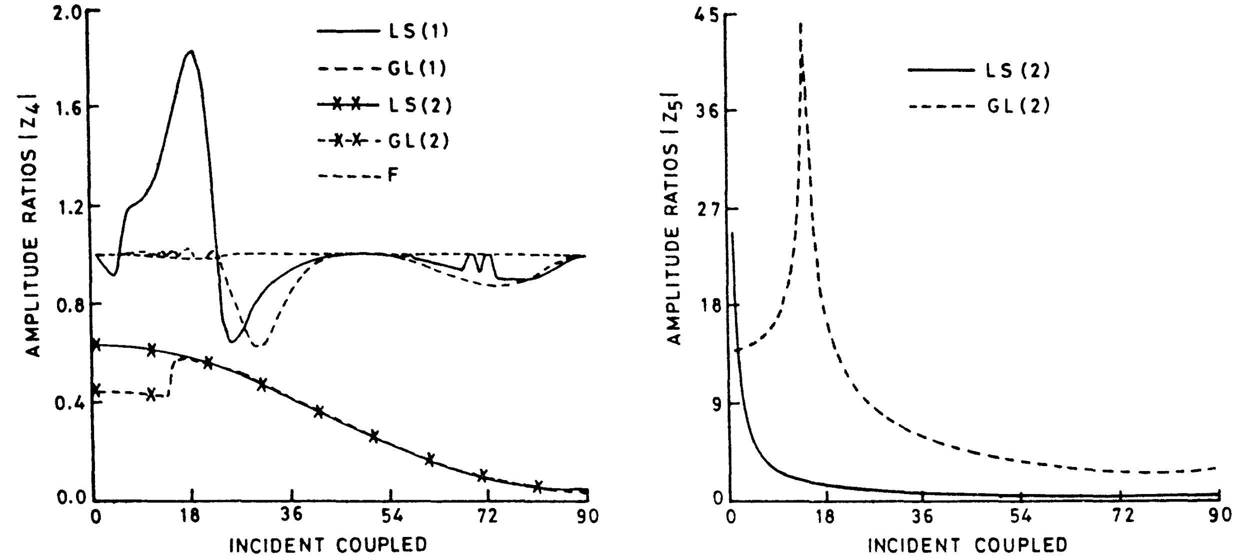



FIG. 10&11. Variations of the amplitude ratios with incident angle of coupled-wave

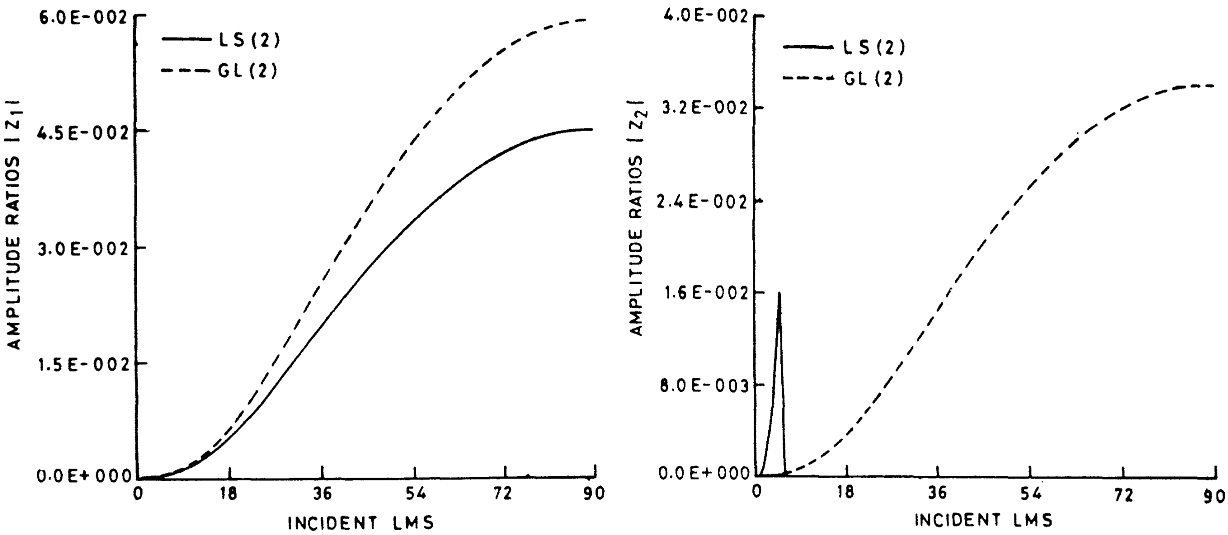



FIG. 12&13. Variations of amplitude ratios with incident angle of L M S-wave

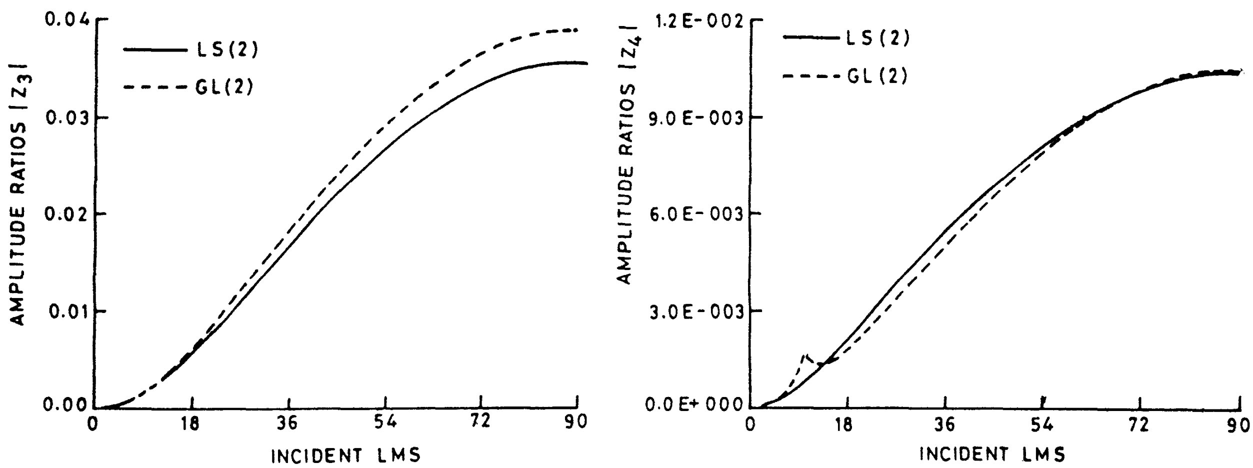



FIG. 14&15. Variations of amplitude ratios with incident angle of L M S-wave coefficients $|z_3|$ and $|z_4|$ for reflected coupled waves have been shown in Figs. 9 and 10 by multiplying their original values by $10^{-2}$ for LS(2) and GL(2) cases respectively. The reflection coefficients $|z_5|$ for reflected longitudinal microstretch wave are also multiplied by $10^{-2}$ and $10^{-1}$ for LS(2) and GL(2) respectively and have been shown in Fig. 11. It is noticed that the stretch effects on reflected coupled waves ($|z_3|$, $|z_4|$) is maximum for LS-theory as compared to that for GL-theory, whereas the stretch effects on reflected thermal and longitudinal displacement wave is maximum for GL-theory as compared with LS-theory. The variations of reflected longitudinal microstretch wave are different in LS- and GL-theories as shown in Fig. 11.

*Case (iii) : Incident Longitudinal Microstretch Wave*

The variations of reflection coefficients $|z_i|$, ($i$ = 1, 2, ..., 5) for various reflected waves for LS(2) and GL(2) cases have been depicted in Figs. 12 to 16. In LS(1), GL(1) and free micropolar elastic (F) cases, these reflected waves do not appear. The reflection coefficient $|z_2|$ of reflected LD-wave for LS(2) case has been multiplied by $10^2$ to its original value and has been shown in Fig. 13. From Figs. 12 to 16, it is clear that the variations of reflected waves are different for both LS- and GL-theories. This difference is minimum when $I = 0^\circ$ and maximum when $I = 90^\circ$.

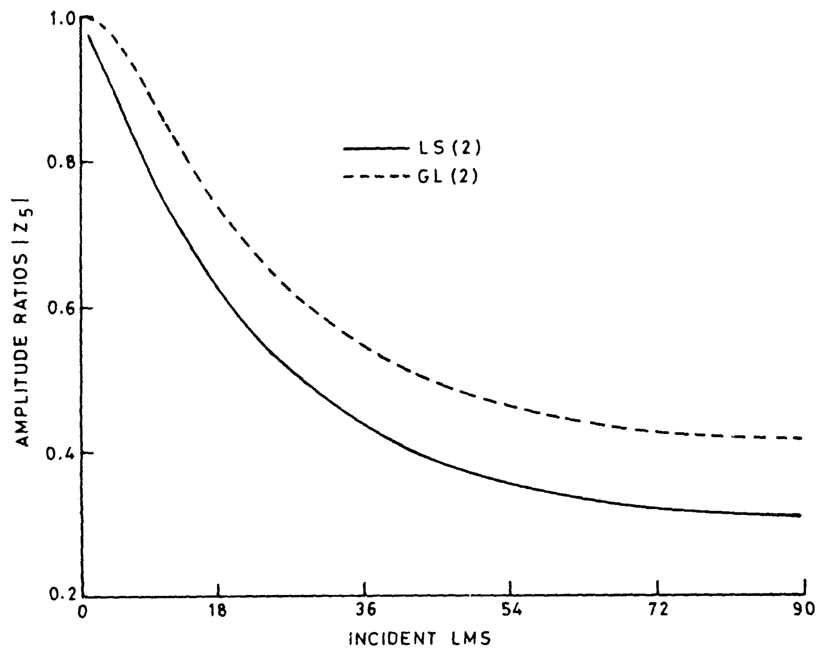



FIG. 16. Variations of amplitude ratios with incident angle of L M S-wave

ACKNOWLEDGEMENTS

One of the authors (Baljeet Singh) is highly grateful to CSIR (New Delhi), India for its kind financial support during this research work.

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